Evaluate $\int_0^{\pi} \tan^2 x \, dx$. **SCORE: _____ / _____ PTS**

$$= \int_0^{\frac{\pi}{2}} \tan^2 x \, dx + \int_{\frac{\pi}{2}}^{\pi} \tan^2 x \, dx$$

$$\int_0^{\frac{\pi}{2}} \tan^2 x \, dx = \lim_{N \to \frac{\pi}{2}^-} \int_0^N \tan^2 x \, dx$$

$$= \lim_{N \to \frac{\pi}{2}^-} \int_0^N (\sec^2 x - 1) \, dx$$

$$= \lim_{N \to \frac{\pi}{2}^-} (\tan x - x) \Big|_0^N$$

$$= \lim_{N \to \frac{\pi}{2}^-} (\tan N - N) \text{ DNE } (\tan N \to \infty)$$

SO $\int_0^{\pi} \tan^2 x \, dx$ DIVERGES

Evaluate $\int_{-\pi}^{\pi} x^5 \cos x \, dx$. **SCORE: _____ / _____ PTS**

$$(-x)^5 \cos(-x) = -x^5 \cos x$$

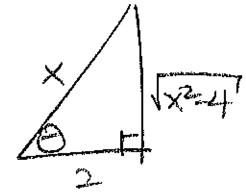
SO $x^5 \cos x$ IS ODD + CONTINUOUS ON $[-\pi, \pi]$

SO $\int_{-\pi}^{\pi} x^5 \cos x \, dx = 0$

Evaluate $\int \frac{x^4}{\sqrt{x^2-4}}\,dx$.

SCORE: _____ / _____ PTS

$$\sec^2\theta - 1 = \tan^2\theta$$
$$4\sec^2\theta - 4 = 4\tan^2\theta$$
$$x^2 - 4 = 4\tan^2\theta$$
$$x = 2\sec\theta \longrightarrow \frac{x}{2} = \sec\theta$$
$$dx = 2\sec\theta\tan\theta\,d\theta$$



$$= \int \frac{16\sec^4\theta}{\cancel{2\tan\theta}}\cdot \cancel{2}\sec\theta\,\cancel{\tan\theta}\,d\theta = \int 16\sec^5\theta\,d\theta$$
$$= 16\left(\tfrac{1}{4}\sec^3\theta\tan\theta + \tfrac{3}{4}\int \sec^3\theta\,d\theta\right)$$
$$= 16\left(\tfrac{1}{4}\sec^3\theta\tan\theta + \tfrac{3}{4}\left(\tfrac{1}{2}\sec\theta\tan\theta + \tfrac{1}{2}\ln|\sec\theta+\tan\theta|\right)\right) + C$$
$$= 4\sec^3\theta\tan\theta + 6\sec\theta\tan\theta + 6\ln|\sec\theta+\tan\theta| + C$$
$$= 4\left(\tfrac{x}{2}\right)^3\frac{\sqrt{x^2-4}}{2} + 6\left(\tfrac{x}{2}\right)\frac{\sqrt{x^2-4}}{2} + 6\ln\left|\frac{x}{2}+\frac{\sqrt{x^2-4}}{2}\right| + C$$
$$= \tfrac{1}{4}x^3\sqrt{x^2-4} + \tfrac{3}{2}x\sqrt{x^2-4} + 6\ln\left|x+\sqrt{x^2-4}\right| + C$$

Evaluate $\int_1^2 \frac{2x^2-16x+30}{x^3-12x^2+45x}\,dx$.

SCORE: _____ / _____ PTS

$$u = x^3 - 12x^2 + 45x$$
$$\frac{du}{dx} = 3x^2 - 24x + 45$$
$$= 3(x^2 - 8x + 15)$$
$$dx = \frac{1}{3(x^2-8x+15)}\,du$$
$$\frac{2(x^2-8x+15)}{x^3-12x^2+45x}\,dx = \frac{2\cancel{(x^2-8x+15)}}{x^3-12x^2+45x}\cdot\frac{1}{3\cancel{(x^2-8x+15)}}\,du$$
$$= \frac{2}{3u}\,du$$
$$\int_{34}^{50}\frac{2}{3u}\,du = \frac{2}{3}\ln|u|\Big|_{34}^{50}$$
$$= \frac{2}{3}(\ln 50 - \ln 34)$$
$$= \frac{2}{3}\ln\frac{25}{17}$$

Evaluate $\int_0^1 \frac{3+\sin x}{x^2}\,dx$.

SCORE: _____ / _____ PTS

$-1 \le \sin x \le 1$

$2 \le 3+\sin x \le 4$

$0 \le \frac{2}{x^2} \le \frac{3+\sin x}{x^2} \le \frac{4}{x^2}$

$\int_0^1 \frac{2}{x^2}\,dx$ DIVERGES SINCE $p=2>1$

SO $\int_0^1 \frac{3+\sin x}{x^2}\,dx$ DIVERGES

Evaluate $\int_0^1 (\arcsin x)^2\,dx$.

SCORE: _____ / _____ PTS

$u = \arcsin x$; $x=1 \rightarrow u=\frac{\pi}{2}$; $x=0 \rightarrow u=0$

$x = \sin u$

$dx = \cos u\,du$

$\int_0^{\frac{\pi}{2}} u^2 \cos u\,du$

$= (u^2 \sin u + 2u\cos u - 2\sin u)\Big|_0^{\frac{\pi}{2}}$

$= \left(\frac{\pi^2}{4} - 2\right) - (0)$

$= \frac{\pi^2}{4} - 2$

f		dg
u^2	$+$	$\cos u$
$2u$	$-$	$\sin u$
2	$+$	$-\cos u$
0		$-\sin u$

Evaluate $\int \frac{30-x}{x^3+2x^2+5x}\,dx$. **SCORE: _____ / _____ PTS**

$$\frac{30-x}{x(x^2+2x+5)} = \frac{A}{x} + \frac{B(2x+2)+C}{x^2+2x+5}$$

$$30 - x = A(x^2+2x+5) + Bx(2x+2) + Cx$$

$$x=0:\ 30 = 5A \longrightarrow A = 6$$

$$x=-1:\ 31 = 4A - C \longrightarrow C = 4A - 31 = -7$$

$$\text{COEF OF } x^2:\ 0 = A + 2B \longrightarrow B = -\tfrac{1}{2}A = -3$$

$$\int \left(\frac{6}{x} - \frac{3(2x+2)}{x^2+2x+5} - \frac{7}{(x+1)^2+2^2} \right) dx$$

$$= 6\ln|x| - 3\ln(x^2+2x+5) - \frac{7}{2}\tan^{-1}\frac{x+1}{2} + C$$